Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Monday 18 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A small ball is moving on a horizontal plane when it strikes a smooth vertical wall. The coefficient of restitution between the ball and the wall is e. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall. Immediately after the impact the direction of motion of the ball makes an angle of 30° with the wall.

(*a*) Find the fraction of the kinetic energy of the ball which is lost in the impact.
(6)

(*b*) Find the value of e.
(4)

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2. A lorry of mass M moves along a straight horizontal road against a constant resistance of magnitude R. The engine of the lorry works at a constant rate RU, where U is a constant. At time t, the lorry is moving with speed v.

(*a*) Show that $Mv\dfrac{\mathrm{d}v}{\mathrm{d}t} = R(U - v)$.
(3)

At time $t = 0$, the lorry has speed $\frac{1}{4}U$ and the time taken by the lorry to attain a speed of $\frac{1}{3}U$ is $\dfrac{kMU}{R}$, where k is a constant.

(*b*) Find the exact value of k.
(7)

3.

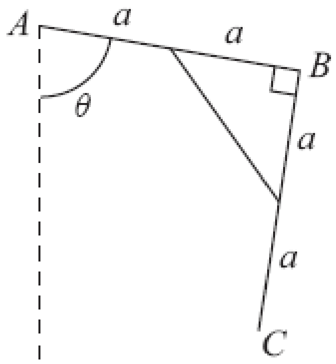

Figure 1

A framework consists of two uniform rods *AB* and *BC*, each of mass *m* and length $2a$, joined at *B*. The mid-points of the rods are joined by a light rod of length $a\sqrt{2}$, so that angle *ABC* is a right angle. The framework is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis passes through the point *A* and is perpendicular to the plane of the framework. The angle between the rod *AB* and the downward vertical is denoted by θ, as shown in Fig. 1.

(*a*) Show that the potential energy of the framework is

$$-mga(3\cos\theta + \sin\theta) + \text{constant}.$$

(4)

(*b*) Find the value of θ when the framework is in equilibrium, with *B* below the level of *A*.

(4)

(*c*) Determine the stability of this position of equilibrium.

(4)

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4. At 12 noon, ship *A* is 20 km from ship *B*, on a bearing of 300°. Ship *A* is moving at a constant speed of 15 km h^{-1} on a bearing of 070°. Ship *B* moves in a straight line with constant speed V km h^{-1} and intercepts *A*.

(*a*) Find, giving your answer to 3 significant figures, the minimum possible for *V*.

(3)

It is now given that $V = 13$.

(*b*) Explain why there are two possible times at which ship *B* can intercept ship *A*.

(2)

(*c*) Find, giving your answer to the nearest minute, the earlier time at which ship *B* can intercept ship *A*.

(8)

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5. A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass m kg. The spheres are moving on a smooth horizontal plane when they collide. At the instant of collision the line joining the centres of the spheres is parallel to $\mathbf{j}$. Immediately **after** the collision, the velocity of A is $(3\mathbf{i} - \mathbf{j})$ m s^{-1} and the velocity of B is $(2\mathbf{i} + \mathbf{j})$ m s^{-1}. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(*a*) Find the velocities of the two spheres immediately before the collision.

(7)

(*b*) Find the magnitude of the impulse in the collision.

(2)

(*c*) Find, to the nearest degree, the angle through which the direction of motion of A is deflected by the collision.

(4)

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6. A small ball is attached to one end of a spring. The ball is modelled as a particle of mass 0.1 kg and the spring is modelled as a light elastic spring AB, of natural length 0.5 m and modulus of elasticity 2.45 N. The particle is attached to the end B of the spring. Initially, at time $t = 0$, the end A is held at rest and the particle hangs at rest in equilibrium below A at the point E. The end A then begins to move along the line of the spring in such a way that, at time t seconds, $t \leq 1$, the downward displacement of A from its initial position is $2 \sin 2t$ metres. At time t seconds, the extension of the spring is x metres and the displacement of the particle below E is y metres.

(*a*) Show, by referring to a simple diagram, that $y + 0.2 = x + 2 \sin 2t$.

(3)

(*b*) Hence show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 49y = 98 \sin 2t$.

(5)

Given that $y = \dfrac{98}{45} \sin 2t$ is a particular integral of this differential equation,

(*c*) find y in terms of t.

(5)

(*d*) Find the time at which the particle first comes to instantaneous rest.

(4)

TOTAL FOR PAPER: 75 MARKS

END